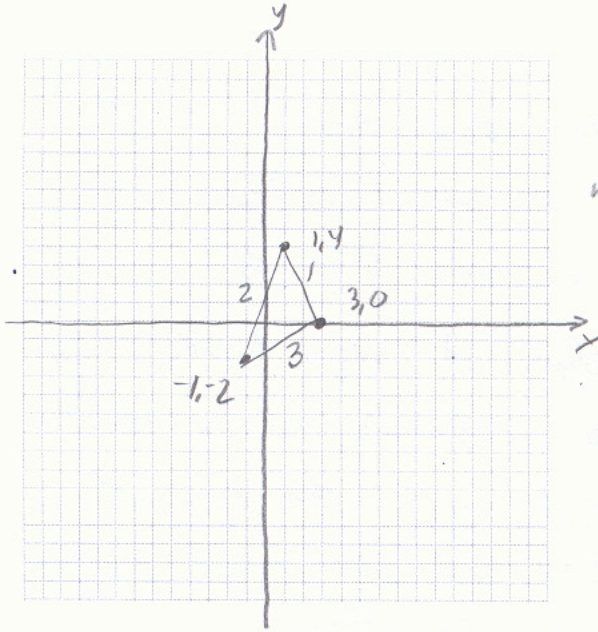
P93 #17



$(1,4)\ (3,0)$

$m = \frac{0-4}{3-1} = \frac{-4}{2} = -2$

SIDE 1

$y - 0 = -2(x-3)$

$y = -2x + 6$ *

$2x + y = 6$ *

$2x + y - 6 = 0$ *

SIDE 2 $(1,4)\ (-1,-2)$

$\frac{4 - -2}{1 - -1} = \frac{6}{2} = 3$

$y - 4 = 3(x-1)$

$y - 4 = 3x - 3$

$y = 3x + 1$ *

$3x - y = -1$ *

$3x - y + 1 = 0$ *

SIDE 3 $(-1,-2)\ (3,0)$

$\frac{0 - -2}{3 - -1} = \frac{2}{4} = \frac{1}{2}$

$y - 0 = \frac{1}{2}(x-3)$

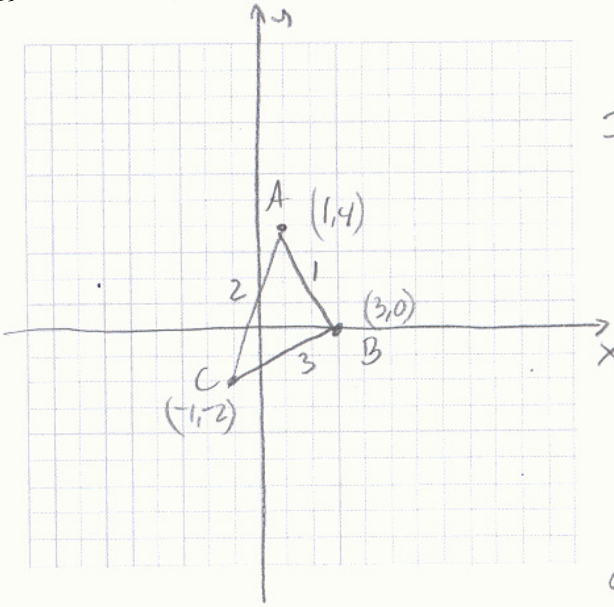
$2\left(y = \frac{1}{2}x - \frac{3}{2}\right)$ *

$2y = x - 3$

$x - 2y = 3$ *

$x - 2y - 3 = 0$ *

p93 #19



SIDE 1

I NOTICE FROM #17:

Slope of side 1 is -2

Slope of side 3 is $\frac{1}{2}$

The slopes of side 1 & side 2 are opposite reciprocals, so the form a right angle. the equations of side 1 and side 2 are the equations for the altitudes.

II We need the equation for the altitude from Pt. B to side 2. Checking the length of the sides (distance formula) shows that both sides are equal.* So we first need to find the midpoint of side 2:

$$M = \left(\frac{1 + -1}{2}, \frac{4 + -2}{2}\right)$$

$$= \left(\frac{0}{2}, \frac{2}{2}\right)$$

$$M = (0, 1)$$

* the length of each side is $\sqrt{20}$

III Now we take the midpt. and the coordinates of Pt. B and find the equation of the altitude:

$$(0,1) \quad (3,0)$$

$$m = \frac{0-1}{3-0} = \frac{-1}{3}$$

$$y - 0 = -\frac{1}{3}(x-3)$$

$$-3\left(y = -\frac{1}{3}x + 1\right)$$

$$-3y = 1x - 3$$

$$x + 3y - 3 = 0$$

(20) For #20 you are given the equations for three lines. Call them A, B, & C. Using the methods from §1.7, find the point of intersection of A & B, then A & C, and then B & C.

(21) The median is the line from a vertex to the midpoint of the opposite side. You first need to find the midpoint of each side, then use these coordinates and the coordinates of the vertex opposite the side to find the equation of the line.

P 93 #20, 21

(20) Find vertices of Δ with sides $\underbrace{x-5y+8=0}_{A}$, $\underbrace{4x-y-6=0}_{B}$, $\underbrace{3x+4y+5=0}_{C}$

P(A,B)

$x-5y+8=0$

$4x-y-6=0$

$x=5y-8$

$4(5y-8)-y-6=0$

$20y-32-y-6=0$

$19y-38=0$

$19y=38$

$y=2$

$x-5(2)+8=0$

$x-10+8=0$

$x-2=0$

$x=2$

(2,2)

$5y=x+8$

$y=\frac{x+8}{5}$

$y=4x-6$

$\boxed{A}$

x	y
-4	$\frac{4}{5}$
-2	$\frac{6}{5}$
0	$\frac{8}{5}$
2	2
4	$\frac{12}{5}$

$\boxed{B}$

x	y
-4	-22
-2	-4
0	-6
2	2
4	10

P(A,C)

$x-5y+8=0$

$3x+4y+5=0$

$5y=x+8$

$y=\frac{x+8}{5}$

$4y=-3x-5$

$y=\frac{-3x-5}{4}$

$3x+4\left(\frac{x+8}{5}\right)+5=0$

$5\left(3x+\frac{4x+32}{5}+5=0\right)$

$15x+4x+32+25=0$

$19x+57=0$

$19x=-57$

$x=-\frac{57}{19}$

$x=-3$

$-3-5y+8=0$

$5y=5$

$y=1$

(-3,1)

P(B,C)

$4x-y-6=0$

$3x+4y+5=0$

$y=4x-6$

$4y=-3x-5$

$y=\frac{-3x-5}{4}$

$3x+4(4x-6)+5=0$

$3x+16x-24+5=0$

$19x-19=0$

$19x=19$

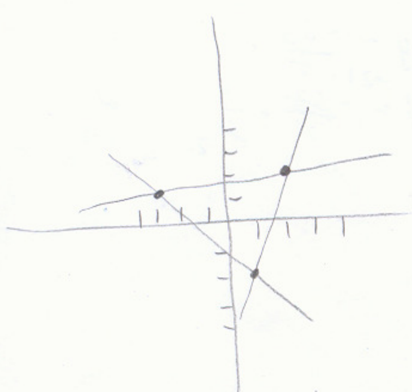
$x=1$

$4(1)-y-6=0$

$-y-2=0$

$y=-2$

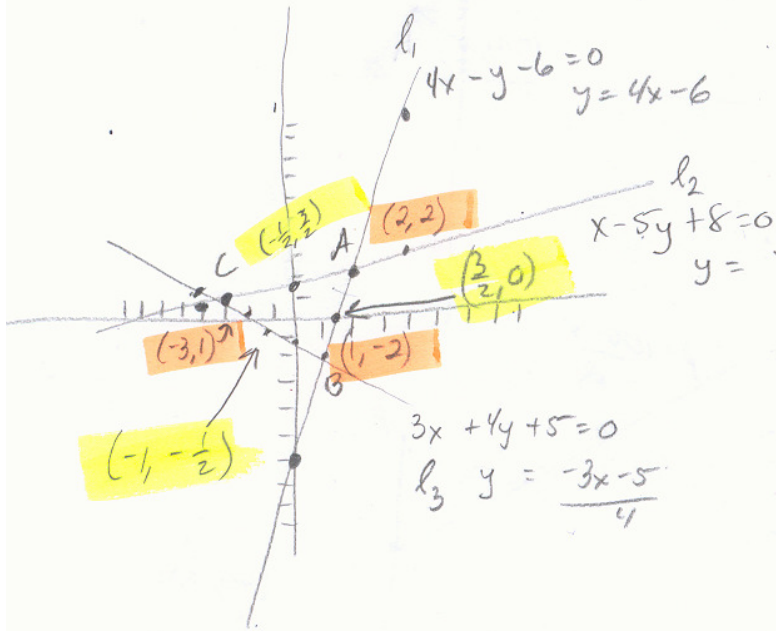
(1,-2)



$\boxed{C}$

x	y
-4	$\frac{7}{4}$
-2	$\frac{1}{4}$
0	$-\frac{5}{4}$
2	$-\frac{11}{4}$
4	$-\frac{17}{4}$
1	-2

(21) Find equations of the medians of the triangle of problem 20



$$\overline{AC} = \sqrt{(2 - -3)^2 + (2-1)^2}$$
$$= \sqrt{25 + 1}$$
$$= \sqrt{26}$$

$$\text{Mdpt}_{\overline{AC}} = \left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$$
$$= \frac{2 + -3}{2}, \frac{2+1}{2}$$
$$= -\frac{1}{2}, \frac{3}{2}$$

$$\text{Mdpt}_{\overline{BC}} = \left(\frac{1+-3}{2}, \frac{-2+1}{2}\right)$$
$$= \left(-\frac{2}{2}, -\frac{1}{2}\right)$$
$$= \left(-1, -\frac{1}{2}\right)$$

$$\text{Mdpt}_{\overline{AB}} = \left(\frac{1+2}{2}, \frac{-2+2}{2}\right)$$
$$= \left(\frac{3}{2}, 0\right)$$

$\left(-\frac{1}{2}, \frac{3}{2}\right)$, $(1, -2)$

$$m = \frac{-2 - \frac{3}{2}}{1 - \left(-\frac{1}{2}\right)} = \frac{-\frac{4}{2} - \frac{3}{2}}{\frac{2}{2} - -\frac{1}{2}}$$
$$= \frac{-\frac{7}{2}}{\frac{3}{2}} = \frac{-7}{3}$$

$$y - y_1 = m(x - x_1)$$
$$y - (-2) = -\frac{7}{3}(x - 1)$$
$$y + 2 = -\frac{7}{3}x + \frac{7}{3}$$
$$y = \frac{-7}{3}x + \frac{7}{3} - \frac{6}{3}$$
$$y = \frac{-7}{3}x + \frac{1}{3}$$
$$3y = -7x + 1$$
$$7x + 3y - 1 = 0$$

$(-3, 1)$ $\left(\frac{3}{2}, 0\right)$

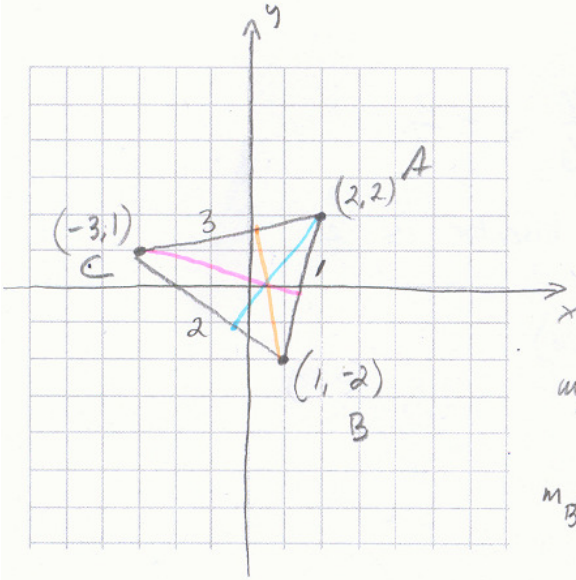
$$m = \frac{0 - 1}{\frac{3}{2} - (-3)}$$
$$= \frac{-1}{\frac{3}{2} + \frac{6}{2}}$$
$$= \frac{-1}{\frac{9}{2}}$$
$$= -\frac{2}{9}$$

$$y - y_1 = m(x - x_1)$$
$$y - 0 = \frac{-2}{9}\left(x - \frac{3}{2}\right)$$
$$y = -\frac{2}{9}x + \frac{1}{3}$$
$$9y = -2x + 3$$
$$2x + 9y - 3 = 0$$

$\left(-1, -\frac{1}{2}\right)$ $(2,2)$

$$m = \frac{2 - -\frac{1}{2}}{2 - -1}$$
$$= \frac{\frac{5}{2}}{3} \quad \frac{5}{2} \cdot \frac{1}{3}$$
$$= \frac{5}{6}$$

$$y - 2 = \frac{5}{6}(x - 2)$$
$$y - 2 = \frac{5}{6}x - \frac{10}{6}$$
$$6y - 12 = 5x - 10$$
$$5x - 6y + 2 = 0$$



I. Find the slopes of the lines.

$\overline{AB}$ is line 1

$\overline{BC}$ is line 2

$\overline{AC}$ is line 3

$$m_{\overline{AB}} = \frac{2 - {}^-2}{2-1} = 4 \qquad m_{\overline{AC}} = \frac{2-1}{2-{}^-3} = \frac{1}{5}$$

$$m_{\overline{BC}} = \frac{{}^-2-1}{1-{}^-3} = \frac{-3}{4}$$

By the slopes we see that there are no right angles.

II An altitude from a vertex forms a right angle with the side opposite. So, starting with A:

A line from A to side 2 must be perpendicular to 2. the slope of side 2 ($\overline{BC}$) is $-\frac{3}{4}$, so the slope of the altitude is $\frac{4}{3}$

Using the slope and Point A:

$m = \frac{4}{3} \qquad P = (2,2)$

$y - 2 = \frac{4}{3}(x - 2)$

$3\left(y - 2 = \frac{4}{3}x - \frac{8}{3}\right)$

$3y - 6 = 4x - 8$

$4x - 3y - 2 = 0$

III. Altitude from B to side 3:

$m_{\overline{AC}} = \frac{1}{5} \qquad P = (1, {}^-2)$

slope of the altitude will be $^-5$

$y - ({}^-2) = {}^-5(x - 1)$

$y + 2 = -5x + 5$

$5x + y - 3 = 0$

IV Altitude from C to side 1:

$m_{\overline{AB}} = 4 \qquad P = (-3, 1)$

slope of the altitude will be $-\frac{1}{4}$

$y - 1 = -\frac{1}{4}(x + 3)$

$4\left(y - 1 = -\frac{1}{4}x - \frac{3}{4}\right)$

$4y - 4 = -x - 3$

$x + 4y - 1 = 0$

P93 #25

$\overline{AB}$ $\overset{A}{(4,2)}$ $\overset{B}{(-2,6)}$

$$M = (x,y) = \left(\frac{4+-2}{2}, \frac{2+6}{2}\right)$$

$$= \left(\frac{2}{2}, \frac{8}{2}\right)$$

$$= (1, 4)$$

the midpoint of $\overline{AB}$ is $P = (1,4)$

$$m_{\overline{AB}} = \frac{6-2}{-2-4} = \frac{4}{-6} = -\frac{2}{3}$$

slope of the $\perp$ bisector is $\frac{3}{2}$

$m = \frac{3}{2}$, $P = (1,4)$

$$y - 4 = \frac{3}{2}(x-1)$$

$$2\left(y - 4 = \frac{3}{2}x - \frac{3}{2}\right)$$

$$2y - 8 = 3x - 3$$

$$3x - 2y + 5 = 0$$

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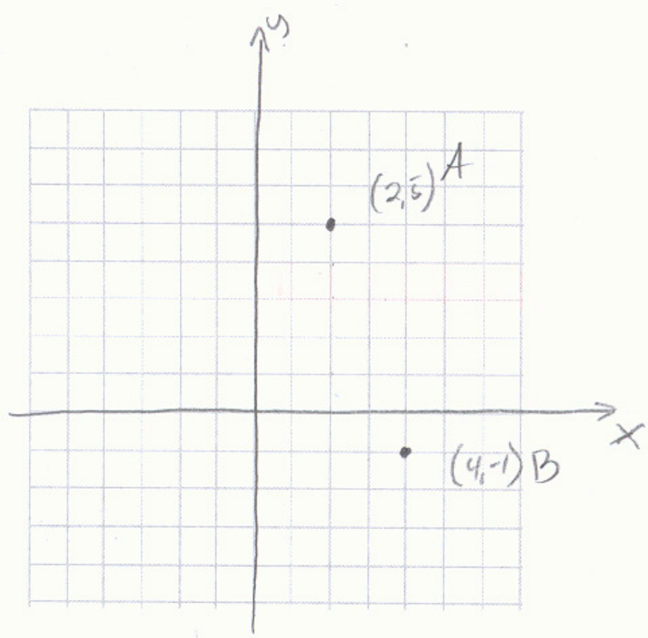


$$m_{\overline{AB}} = \frac{5- -1}{2-4} = \frac{6}{-2} = -3$$

$$M_{\overline{AB}} = \left(\frac{2+4}{2}, \frac{5+-1}{2}\right)$$

$$= (3, 2)$$

$m = \frac{1}{3}$, $P = (3,2)$

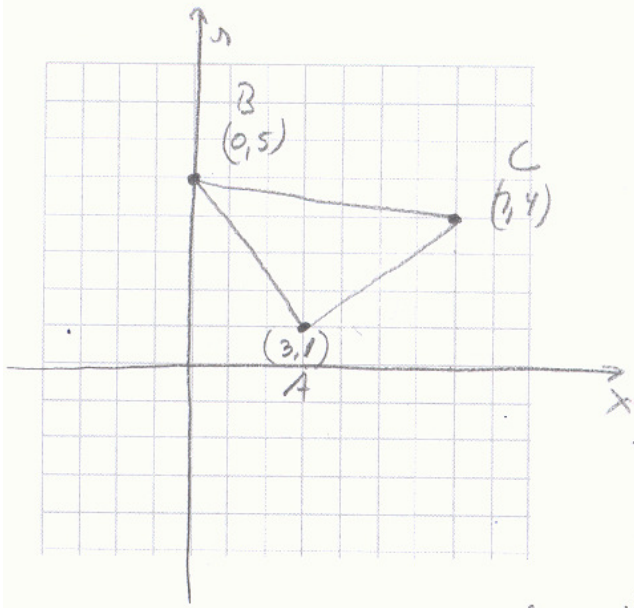
$$y - 2 = \frac{1}{3}(x-3)$$

$$3\left(y - 2 = \frac{1}{3}x - 1\right)$$

$$3y - 6 = x - 3$$

$$x - 3y + 3 = 0$$

In order for a point to be equidistant from A & B it must be on the $\perp$ bisector of $\overline{AB}$. The equation for the bisector is $x - 3y + 3 = 0$



$m_{\overline{AB}} = \frac{1-5}{3-0} = \frac{-4}{3}$

$m_{\overline{BC}} = \frac{5-4}{0-7} = \frac{1}{-7}$

$m_{\overline{AC}} = \frac{4-1}{7-3} = \frac{3}{4}$

Right Δ

$d_{\overline{AB}} = \sqrt{(3-0)^2 + (1-5)^2}$
$= \sqrt{9 + 16}$
$= \sqrt{25}$
$= 5$

$d_{\overline{AC}} = \sqrt{(7-3)^2 + (4-1)^2}$
$= \sqrt{16 + 9}$
$= \sqrt{25}$
$= 5$

I Altitude:

slope is $\perp$ to $\overline{BC}$

slope $= 7$

$m = 7 \quad A = (3,1)$

$y - 1 = 7(x-3)$

$y - 1 = 7x - 21$

$7x - y - 20 = 0$

II median:

$M_{\overline{BC}} = \left(\frac{0+7}{2}, \frac{5+4}{2}\right)$

$= \left(\frac{7}{2}, \frac{9}{2}\right) \quad A = (3,1)$

$m = \frac{1 - \frac{9}{2}}{3 - \frac{7}{2}} = \frac{\frac{2}{2} - \frac{9}{2}}{\frac{6}{2} - \frac{7}{2}} = \frac{-\frac{7}{2}}{-\frac{1}{2}} = -\frac{7}{\cancel{2}} \cdot -\frac{\cancel{2}}{1} = 7$

$m = 7 \quad P = (3,1)$

$7x - y - 20 = 0$

the equations for the median and altitude are the same, so the triangle is an isosceles.