Algebra 1
Unit 6: Measurement

Time Frame: Approximately three weeks

Unit Description

This unit is an advanced study of measurement. It includes the topics of precision and accuracy and investigates the relationship between the two. The investigation of absolute and relative error and how they each relate to measurement is included. Significant digits are also studied as well as how computations on measurements are affected when considering precision and significant digits.

Student Understandings

Students should be able to find the precision of an instrument and determine the accuracy of a given measurement. They should know the difference between precision and accuracy. Students should see error as the uncertainty approximated by an interval around the true measurement. They should be able to calculate and use significant digits to solve problems.

Guiding Questions

1. Can students determine the precision of a given measurement instrument?
2. Can students determine the accuracy of a measurement?
3. Can students differentiate between what it means to be precise and what it means to be accurate?
4. Can students discuss the nature of precision and accuracy in measurement and note the differences in final measurement values that may result from error?
5. Can students calculate using significant digits?

Unit 6 Grade-Level Expectations (GLEs)

GLE #	GLE Text and Benchmarks
Number and Number Relations	
4.	Distinguish between an exact and an approximate answer, and recognize errors introduced by the use of approximate numbers with technology (N-3-H) (N-4-H) (N-7-H)
5.	Demonstrate computational fluency with all rational numbers (e.g., estimation, mental math, technology, paper/pencil) (N-5-H)

Measurement	
17.	Distinguish between precision and accuracy (M-1-H)
GLE #	**GLE Text and Benchmarks**
18.	Demonstrate and explain how the scale of a measuring instrument determines the precision of that instrument (M-1-H)
19.	Use significant digits in computational problems (M-1-H) (N-2-H)
20.	Demonstrate and explain how relative measurement error is compounded when determining absolute error (M-1-H) (M-2-H) (M-3-H)
21.	Determine appropriate units and scales to use when solving measurement problems (M-2-H) (M-3-H) (M-1-H)

Sample Activities

Activity 1: What Does it Mean to be Accurate? (GLEs: 4, 17)

Materials List: paper, pencil, three or more different types of scales from science department, three or more different bathroom scales, student's watches, Internet access, What Does It Mean To Be Accurate? BLM, sticky notes

This unit on measurement will have many new terms to which students have not yet been exposed. Have students maintain a *vocabulary self-awareness chart* (view literacy strategy descriptions) for this unit. *Vocabulary self-awareness* is valuable because it highlights students' understanding of what they know, as well as what they still need to learn, in order to fully comprehend the concept. Students indicate their understanding of a term/concept, but then adjust or change the marking to reflect their change in understanding. The objective is to have all terms marked with a + at the end of the unit. A sample chart is shown below.

Word	**+**	**☑**	**-**	**Example**	**Definition**
accuracy					
precision					
Relative error					
Absolute error					
Significant digits					

Be sure to allow students to revisit their self-awareness charts often to monitor their developing knowledge about important concepts. Sample terms to use include accuracy, precision, significant digits, absolute error, and relative error.

Have students use the What Does It Mean To Be Accurate? BLM to complete this activity.
Talk with students about the meaning of "accuracy" in measurement. Accuracy indicates how close a measurement is to the accepted "true" value. For example, a scale is expected to read 100 grams if a standard 100 gram weight is placed on it. If the scale does not read 100 grams, then the scale is said to be inaccurate. If possible, obtain a standard weight from one of the science teachers along with several scales. With students, determine which scale is closest to the known value and use this information to determine which scale is most accurate.

Next, ask students if they have ever weighed themselves on different scales—if possible, provide different scales for students to weigh themselves. The weight measured for a person might vary according to the accuracy of the instruments being used. Unless "true" weight is known (i.e., there is a known standard to judge each scale), it cannot be determined which scale is most accurate. Generally, when a scale or any other measuring device is used, the readout is automatically accepted without really thinking about its validity. People do this without knowing if the tool is giving an accurate measurement. Also, modern digital instruments convey such an aura of accuracy and reliability (due to all the digits it might display) that this basic rule is forgotten—there is no such thing as a perfect measurement. Digital equipment does not guarantee 100% accuracy. Note: If some students object to being weighed, students might weigh their book bags or other fairly heavy items. Adjust the BLM if this is done.

Have all of the students who have watches record the time (to the nearest second) at the same moment and hand in their results. Post the results on the board or overhead—there should be a wide range of answers. Ask students, "Which watch is the most accurate?" Students should see that in order to make this determination, the true time must be known. Official time in the United States is kept by NIST and the United States Naval Observatory, which averages readings from the 60 atomic clocks it owns. Both organizations also contribute to UTC, the world universal time. The website http://www.time.gov has the official U.S. time, but even its time is "accurate to within .7 seconds." Cite this time at the same time the students are determining the time from their watches to see who has the most accurate time.

Lead students in a discussion as to why their watches have different times (set to home, work, and so on) and how their skill at taking a reading on command might produce different readings on watches that may be set to the same time.

Ultimately, students need to understand that accuracy is really a measure of how close a measurement is to the "true" value. Unless the true value is known, the accuracy of a measurement cannot be determined.

Activity 2: How Precise is Your Measurement Tool? (GLEs: 4, 17, 18)

Materials List: paper, pencil, rulers with different subdivisions, four-sided meter sticks, toothpicks, What is Precision? BLM, wall chart , blue masking tape

Discuss the term "precision" with the class. Precision is generally referred to in one of two ways. It can refer to the degree to which repeated readings on the same quantity agree with each other. We will study this definition in Activity 4.

Have students use the What is Precision? BLM for this activity.

Precision can also be referred to in terms of the unit used to measure an object. Precision depends on the refinement of the measuring tool. Help students to understand that no measurement is perfect. When making a measurement, scientists give their best estimate of the true value of a measurement, along with its uncertainty.

The precision of an instrument reflects the number of digits in a reading taken from it—the degree of refinement of a measurement. Discuss with students the degree of precision with which a measurement can be made using a particular measurement tool. For example, have on hand different types of rulers (some measuring to the nearest inch, nearest $\frac{1}{2}$ inch, nearest $\frac{1}{4}$ inch, nearest $\frac{1}{8}$ inch, nearest $\frac{1}{16}$ inch, nearest centimeter, and nearest millimeter) and discuss with students which tool would give the most precise measurement for the length of a particular item (such as the length of a toothpick). Have students record measurements they obtain with each type of ruler and discuss their findings.

Divide students into groups. Supply each group with a four-sided meter stick. (This meter stick is prism-shaped with different divisions of a meter on each side. The meter stick can be purchased at www.boreal.com, NASCO, and other suppliers.)

Cover the side of the meter stick that has no subdivisions with two strips of masking tape and label it as side 1. (You need two layers of masking tape so the markings on the meter stick will not show through the tape. The blue tape works better as the darker color prevents markings from showing through better.) Repeat this with the other sides of the stick such that side 2 has decimeter markings, side 3 has centimeter markings, and side 4 has millimeter markings. Have students remove the tape from side 1 and measure the length of a sheet of paper with that side and record their answers. Repeat with the other sides of the meter stick in numerical order. Post a wall chart similar to the one below and have each group record their measurements:

Length of Paper				
	Side 1	Side 2	Side 3	Side 4
Group 1				
Group 2				
Group 3				

Group 4				
Group 5				
Group 6				
Average				

Have students calculate the averages of each column. Lead students to discover that the measurements become closer to the average with the increase in divisions of the meter stick.

Help students understand that the ruler with the greatest number of subdivisions per unit will provide the most precise measure.

Have students complete the following *RAFT writing* assignment (view literacy strategy descriptions) in order to give students a creative format for demonstrating their understanding of precise measurement.

Role- millimeter ruler
Audience-decimeter ruler
Format-advertisement
Topic-Buy my subdivisions

Once *RAFT writing* is completed, have students share with a partner, in small groups, or with the whole class. Students should listen for accurate information and sound logic in the *RAFT*s.

Activity 3: Temperature—How Precise Can You Be? (GLEs: 4, 17, 18)

Materials List: paper, pencil, thermometers

Have students get in groups of three. Provide each team with a thermometer that is calibrated in both Celsius and Fahrenheit. Have each team record the room temperature in both ^{o}C and ^{o}F. Have students note the measurement increments of the thermometer (whether it measures whole degrees, tenths of a degree, and so on) on both scales. Make a class table of the temperatures read by each team. Ask students if it is possible to have an answer in tenths of a degree using their thermometers and why or why not? It is important that students understand that the precision of the instrument depends on the smallest division of a unit on a scale. If the thermometer only has whole degree marks, then it can only be precise to one degree. If the thermometer has each degree separated into tenths of a degree then the measurement is precise to the nearest tenth of a degree. Regardless of the measurement tool being used, this idea of the precision of the instrument holds true.

Activity 4: Repeatability and Precision (GLE: 17)

Materials List: paper, pencil

As stated in Activity 2, precision can also refer to the degree to which repeated readings on the same quantity agree with each other.

Present students with the following situations:

> Jamaal made five different measurements of the solubility of nickel (II) chloride in grams per deciliter of water and obtained values of 35.11, 35.05, 34.98, 35.13, and 35.09 g/dL.
>
> Juanita made five different measurements of the solubility of nickel (II) chloride in grams per deciliter of water and obtained values of 34.89, 35.01, 35.20, 35.11, and 35.13 g/dL.

Have students work with a partner to discuss ways to determine which set of measurements is more precise.

Have students come up with a method for determining which set of measurements is the most precise. Lead students to the determination that the set that has the smallest range is a more precise set of measurements.

Provide students with additional measurement situations so that they have the opportunity to practice determining the more precise set of measurements when given a group of measurements.

Activity 5: Precision vs. Accuracy (GLE: 17)

Materials List: paper, pencil, Target BLM transparency, Precision vs. Accuracy BLM, sticky notes

Student Questions for Purposeful Learning or *SQPL* (view literacy strategy descriptions) is a strategy designed to gain and hold students' interest in the material by having them ask and answer their own questions. Before beginning the activity, place the following statement on the board:

> Accuracy is telling the truth. Precision is telling the same story over and over again.

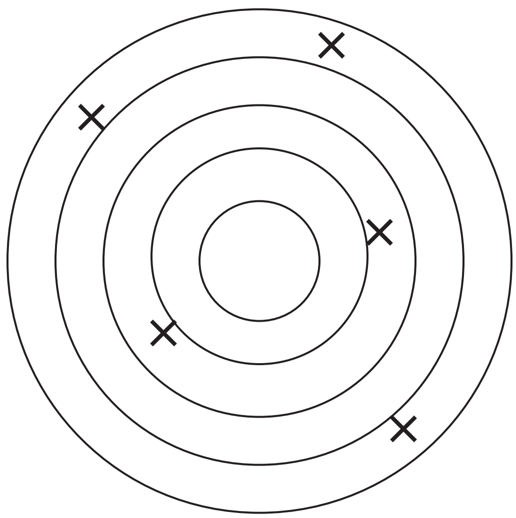
Have students pair up and, based on the statement, generate two or three questions they would like answered. Ask someone from each team to share questions with the whole class and write those questions on the board. As the content is covered in the activity, stop periodically and have students discuss with their partners which questions could be

answered, and have them share answers with the class. Have them record the information in their notebooks.

Create a transparency of the Target BLM which includes the target examples shown below and have students determine if the patterns are examples of precision, accuracy, neither or both. Cover boxed descriptions with sticky notes and remove as the lesson progresses. After the lesson provide students with Target BLM to include in their notes.

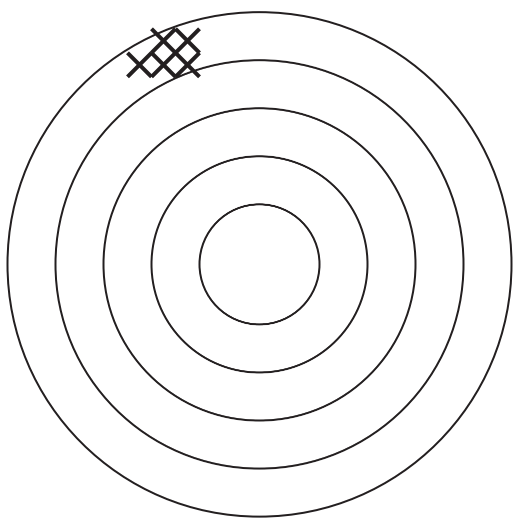
If you were trying to hit a bull's eye (the center of the target) with each of five darts, you might get results such as in the models below. Determine if the results are precise, accurate, neither or both.

Neither Precise Nor Accurate



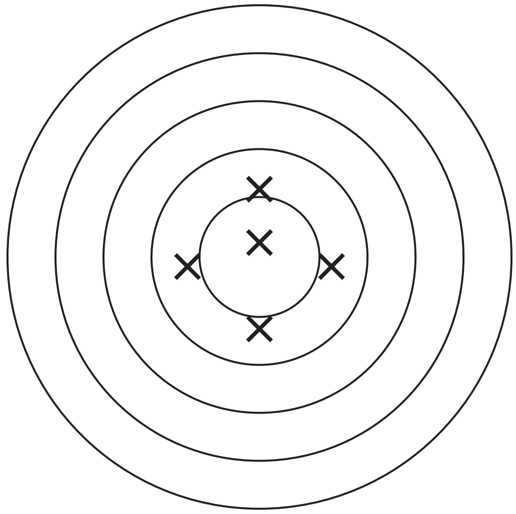
This is a random-like pattern, neither precise nor accurate. The darts are not clustered together and are not near the bull's eye.

Precise, Not Accurate



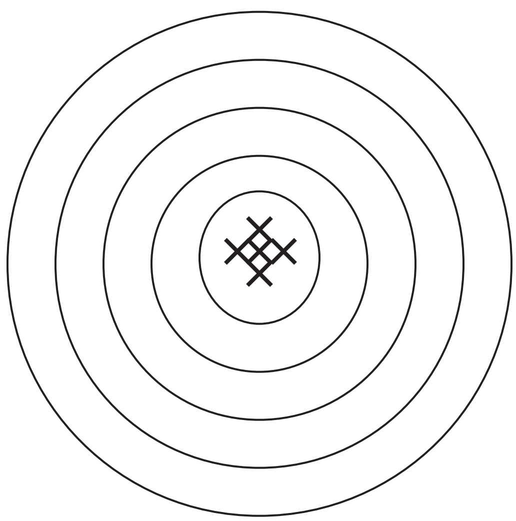
This is a precise pattern, but not accurate. The darts are clustered together but did not hit the intended mark.

Accurate, Not Precise



This is an accurate pattern, but not precise. The darts are not clustered, but their average position is the center of the bull's eye.

Precise and Accurate



This pattern is both precise and accurate. The darts are tightly clustered, and their average position is the center of the bull's eye.

Lead a class discussion reviewing the definitions of precision and accuracy and revisit the class-generated questions.

Use the Precision vs. Accuracy BLM and present the examples to students. Lead a class discussion using the questions on the BLM.

Provide students with more opportunities for practice in determining the precision and/or accuracy of data sets.

Activity 6: Absolute Error (GLEs: 18, <u>20</u>)

Materials List: paper, pencil, Absolute Error BLM, three different scales, 2 different beakers, measuring cup, meter stick, 2 different rulers, calculator, cell phone, wrist watch

In any lab experiment, there will be a certain amount of error associated with the calculations. For example, a student may conduct an experiment to find the specific heat capacity of a certain metal. The difference between the experimental result and the actual (known) value of the specific heat capacity is called absolute error. The formula for calculating absolute error is as follows:

$$\text{Absolute Error} = \left|\text{Observed Value - Actual Value}\right|$$

Review absolute value with students and explain to them that since the absolute value of the difference is taken, the order of the subtraction will not matter.

Present the following problems to students for a class discussion:

Luis measures his pencil and he gets a measurement of 12.8 cm but the actual measurement is 12.5 cm. What is the absolute error of his measurement?
($\text{Absolute Error} = \left|12.8 - 12.5\right| = \left|.3\right| = .3\ cm$)

A student experimentally determines the specific heat of copper to be 0.3897 ^{o}C. Calculate the student's absolute error if the accepted value for the specific heat of copper is 0.38452 ^{o}C. ($\text{Absolute Error} = \left|.3897 - .38452\right| = \left|0.00518\right| = 0.00518$)

Place students in groups and have them rotate through measurement stations. Have students use the Absolute Error BLM to record the data. After students have completed collecting the measurements, present them with information about the actual value of the measurement. Have students calculate the absolute error of each of their measurements.

Examples of stations:

Station	Measurement	Instruments	Actual Value
1	Mass	3 different scales	100 gram weight
2	Volume	2 different sized beakers and a measuring cup	Teacher measured volume of water
3	Length	Meter stick, rulers with 2 different intervals	Sheet of paper
4	Time	Wrist watch, calculator, cell phone	http://www.time.gov

Activity 7: Relative Error (GLEs: 4, 5, 20)

Materials List: paper, pencil

Although absolute error is a useful calculation to demonstrate the accuracy of a measurement, another indication is called relative error. In some cases, a very tiny absolute error can be very significant, while in others, a large absolute error can be relatively insignificant. It is often more useful to report accuracy in terms of relative error. Relative error is a comparative measure. The formula for relative error is as follows:

$$Relative\ Error = \frac{Absolute\ Error}{Actual\ value} \times 100$$

To begin a discussion of absolute error, present the following problem to students:

> Jeremy ordered a truckload of dirt to fill in some holes in his yard. The company told him that one load of dirt is 5 tons. The company actually delivered 4.955 tons.
>
> Chanelle wants to fill in a flowerbed in her yard. She buys a 50-lb bag of soil at a gardening store. When she gets home she finds the contents of the bag actually weigh 49.955 lbs.
>
> Which error is bigger?
>
> *The relative error for Jeremy is 0.9%. The relative error for Chanelle is 0.09%. This tells you that measurement error is more significant for Jeremy's purchase.*

Use these examples to discuss with students the calculation of relative error and how it relates to the absolute error and the actual value of measurement. Explain to students that the relative error of a measurement increases depending on the absolute error *and* the actual value of the measurement.

Provide students with an additional example:

> In an experiment to measure the acceleration due to gravity, Ronald's group calculated it to be 9.96 m/s^2. The accepted value for the acceleration due to gravity is 9.81 m/s^2. Find the absolute error and the relative error of the group's calculation. (*Absolute error is .15 m/s^2, relative error is 1.529%.)*

Provide students with more opportunity for practice with calculating absolute and relative error.

Activity 8: What's the Cost of Those Bananas? (GLEs: 4, 17, 18)

Materials List: paper, pencil, pan scale, electronic scale, fruits or vegetables to weigh

The following activity can be completed as described below if the activity seems reasonable for the students involved. If not, the same activity can be done if there is access to a pan scale and an electronic balance. If done in the classroom, provide items for students to measure—bunch of bananas, two or three potatoes, or other items that will not deteriorate too fast.

Have the students go to the local supermarket and select one item from the produce department that is paid for by weight. Have them calculate the cost of the object using the hanging pan scale present in the department. Record their data. At the checkout counter, have students record the weight given on the electronic scale used by the checker. Have students record the cost of the item. How do the two measurements and costs compare? Have students explain the significance of the number of digits (precision) of the scales and the effect upon cost.

Activity 9: What are Significant Digits? (GLEs: 4, 19)

Materials List: paper, pencil

Discuss with students what significant digits are and how they are used in measurement. Significant digits are defined as all the digits in a measurement one is certain of plus the first uncertain digit. Significant digits are used because all instruments have limits, and there is a limit to the number of digits with which results are reported. Demonstrate and discuss the process of measuring using significant digits.

After students have an understanding of the definition of significant digits, discuss and demonstrate the process of determining the number of significant digits in a number. Explain to students that it is necessary to know how to determine the significant digits so that when performing calculations with numbers they will understand how to state the answer in the correct number of significant digits.

Rules For Significant Digits

1. Digits from 1-9 are always significant.
2. Zeros between two other significant digits are always significant
3. One or more additional zeros to the right of both the decimal place and another significant digit are significant.
4. Zeros used solely for spacing the decimal point (placeholders) are not significant.

Using a chemistry textbook as a resource, provide problems for students to practice in determining the number of significant digits in a measurement.

In their math *learning logs* (view literacy strategy descriptions) have students respond to the following prompt:

Explain the following statement:

> The more significant digits there are in a measurement, the more precise the measurement is.

Allow students to share their entries with the entire class. Have the class discuss the entries to determine if the information given is correct.

Activity 10: Calculating with Significant Digits (GLEs: 4, 19)

Materials List: paper, pencil,

Discuss with students how to use significant digits when making calculations. There are different rules for how to round calculations in measurement depending on whether the operations involve addition/subtraction or multiplication/division. When adding, such as in finding the perimeter, the answer can be no more PRECISE than the least precise measurement (i.e., the perimeter must be rounded to the same decimal place as the least precise measurement). If one of the measures is 15 ft and another is 12.8 ft, then the perimeter of a rectangle (55.6 ft) would need to be rounded to the nearest whole number (56 ft). We cannot assume that the 15 foot measure was also made to the nearest tenth based on the information we have. The same rule applies should the difference between the two measures be needed.

When multiplying, such as in finding the area of the rectangle, the answer must have the same number of *significant digits* as the measurement with the fewest number of significant digits. There are two significant digits in 15 so the area of 192 square feet, would be given as 190 square feet. The same rule applies for division.

Have students find the area and perimeter for another rectangle whose sides measure 9.7 cm and 4.2 cm. The calculated area is (9.7cm)(4.2cm) = 40.74 sq. cm, but should be rounded to 41 sq cm (two significant digits). The perimeter of 27.8 cm would not need to be rounded because both lengths are to the same precision (tenth of a cm).

After fully discussing calculating with significant figures, have students work computational problems (finding area, perimeter, circumference of 2-D figures) dealing with the topic of calculating with significant digits. A chemistry textbook is an excellent source for finding problems of calculations using significant digits.

Activity 11: Measuring the Utilities You Use (GLE: 19)

Materials List: paper, pencil, utility meters around students' households, utility bills

Have students find the various utility meters (water, electricity) for their households. Have them record the units and the number of places found on each meter. Have the class get a copy of their family's last utility bill for each meter they checked. Have students answer the following questions: What units and number of significant digits are shown on the bill? Are they the same? Why or why not? Does your family pay the actual "true value" of the utility used or an estimate? If students do not have access to such information, produce sample drawings of meters used in the community and samples of utility bills so that the remainder of the activity can be completed.

Activity 12: Which Unit of Measurement? (GLEs: 5, 21)

Materials List: paper, pencil, centimeter ruler, meter stick, ounce scale, bathroom scale, quarter, cup, gallon jug, bucket, water

Divide students into groups. Provide students with a centimeter ruler and have them measure the classroom and calculate the area of the room in centimeters. Then provide them with a meter stick and have them calculate the area of the room in meters. Discuss with students which unit of measure was most appropriate to use in their calculations. Ask students if they were asked to find the area of the school parking lot, which unit would they definitely want to use. What about their entire town? In that case, kilometers would probably be better to use. Provide opportunities for discussion and/or examples of measurements of mass (weigh a quarter on a bathroom scale or a food scale) and volume (fill a large bucket with water using a cup or a gallon jug) similar to the linear example of the area of the room. Use concrete examples for students to visually explore the most appropriate units and scales to use when solving measurement problems.

Sample Assessments

General Assessments

- Portfolio Assessment: The student will create a portfolio divided into the following sections:
    1. Accuracy
    2. Precision
    3. Precision vs. Accuracy
    4. Absolute error
    5. Relative error
    6. Significant digits

In each section of the portfolio, the student will include an explanation of each, examples of each, artifacts that were used during the activity, and sample questions given during class. The portfolio will be used as an opportunity for students to demonstrate a true conceptual understanding of each concept.

- The student will complete entries in their math *learning logs* using such topics as these:
  - Darla measured the length of a book to be $11\frac{1}{4}$ inches with her ruler and $11\frac{1}{2}$ inches with her teacher's ruler. Can Darla tell which measurement is more accurate? Why or why not? *(She cannot tell unless she knows which ruler is closer to the actual standard measure)*
  - What does it mean to be precise? Give examples to support your explanation.
  - What is the difference between being precise and being accurate? Explain your answer.
  - When would it be important to measure something to three or more significant digits? Explain your answer.

Activity-Specific Assessments

- <u>Activity 1</u>: The student will write a paragraph explaining in his/her own words what it means to be accurate. He/she will give an example of a real-life situation in which a measurement taken may not be accurate.

- <u>Activity 7</u>: The student will solve sample test questions, such as this:
  Raoul measured the length of a wooden board that he wants to use to build a ramp. He measured the length to be 4.2 m. but his dad told him that the board was actually 4.3 m. His friend, Cassandra, measured a piece of molding to decorate the ramp. Her measurement was .25 m but the actual measurement was .35. Use relative error to determine whose measurement was more accurate. Justify your answer.

- <u>Activity 12</u>: The student will be able to determine the most appropriate unit and/or instrument to use in both English and Metric units when given examples such as:
  How much water a pan holds
  Weight of a crate of apples
  Distance from New Orleans to Baton Rouge
  How long it takes to run a mile
  Length of a room
  Weight of a Boeing 727
  Weight of a t-bone steak
  Thickness of a pencil
  Weight of a slice of bread