

Measurement Error and Units

No measuring tool is perfect. There will always be a difference between the measured quantity and its true value. This is called **measurement error**.

There are two types of measurement error:
absolute error and **relative error**.

Absolute error: the absolute value of the difference between the measured value and the true value in terms of the units of measurement. (inches, feet, pounds, etc.)

Relative error: measures the same difference as a %

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For example, a measurement might be given as 8 cm \pm 0.5 cm. This **does not** mean that the measurement **is** off by 0.5 cm, only that it **could** be. The range of possible measurements is 7.5 cm to 8.5 cm.

To find relative error, divide the absolute error by the initial measurement and multiply by 100.

In this example, $\frac{0.5}{8} \times 100 = 0.0625 \times 100 = 6.25\%$

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Example 1

Eli weighs a bag of flour. The bag weighs 4.8 kg and Eli estimates the absolute error to be 0.4 kg. Give a range of possible weights for the bag and calculate the relative error.

Step 1

Calculate the range.

$$4.8 \text{ kg} - 0.4 \text{ kg} = 4.4 \text{ kg}$$

$$4.8 \text{ kg} + 0.4 \text{ kg} = 5.2 \text{ kg}$$

The range of possible values is 4.4 kg to 5.2 kg.
The relative error is 8.3%

Step 2

Calculate the relative error.

$$\text{relative error} = \frac{\text{absolute error}}{\text{measured value}} \times 100$$

$$\text{relative error} = \frac{0.4}{4.8} \times 100 = 0.083333... \times 100 = 8.3\%$$

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When measurements with errors are **combined** their errors must be combined as well.

When **combining** using addition or subtraction, the absolute error of the final answer is the sum of the absolute errors of each term.

When **combining** using multiplication or division, the relative error of the final answer is the sum of the relative errors of each term.

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Example 2

Gerry lays two pieces of wood end to end. The first piece measures $22'' \pm 0.50''$. The second piece measures $41'' \pm 1.25''$. Give a range of possible values for the total length of the two pieces.

Step 1. Add the measurements.

$$22 + 41 = 63''$$

Step 2. Add the absolute errors.

$22'' \pm 0.50''$. The absolute error is $0.50''$.

$41'' \pm 1.25''$. The absolute error is $1.25''$.

$$0.50 + 1.25 = 1.75''$$

Step 3. Calculate the range of values.

$$63 - 1.75 = 61.25''$$

$$63 + 1.75 = 64.75''$$

Range of values for total length is $61.25''$ to $64.75'' \pm 1.75''$

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EXAMPLE 3

We can find area of a rectangular window by multiplying its _____ by its _____.

The length of a window is measured to be 1.5 meters \pm 0.03 meter and the width to be 1.2 meters \pm 0.03 meter.

Calculate the area of the window. Include the absolute error and the appropriate units.

STEP 1 Calculate the relative errors.

$$\text{relative error} = \frac{\text{absolute error}}{\text{measured value}} \times 100$$

For the length, relative error = $\frac{\quad}{\quad} \times 100 = \frac{\quad}{\quad} \%$

For the width, relative error = $\frac{\quad}{\quad} \times 100 = \frac{\quad}{\quad} \%$

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STEP 2 Find the area of the window.

The units of the answer are

_____ X _____ = _____ or m².

Multiplying the measurements gives

_____ X _____ = _____ m².

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STEP 3 Combine the errors.

When measurements are multiplied, their relative errors

are _____.

The total relative error is _____ + _____ = _____ %.

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STEP 4 Calculate the absolute error.

$$\text{relative error} = \frac{\quad}{\quad} \times 100$$

$$\frac{\quad}{\quad} \% = \frac{\text{absolute error}}{\quad} \times 100$$

$$\text{absolute error} = \frac{\quad}{\quad} \text{ m.}$$

Measurement Error and Units

John measures a distance of 56 yds \pm 2yds. John's friend Carol runs the distance in 8 seconds \pm 0.4 seconds. To find Carol's average speed divide the distance she travels by the time it takes her. Calculate Carol's Average speed with appropriate units and relative error.

THINKING IT THROUGH

What is the relative error for the distance? _____ = _____

What is the relative error for the time? _____ = _____

Divide the distance by the time: _____

What are the units of the answer? _____

How do you combine the relative errors? _____

The total relative error is _____ or _____ %

Carol's average speed is _____ with relative error _____.

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