Honors Analytical Geometry
Review Sheet
Ch 3.3 & 3.4

Name: _____
Date: _____
Period: _____

1. Find the distance from the given line to the point.

a. $x + 2y - 6 = 0$ $(2, -2)$

$d = \frac{|Ax + By + C|}{\sqrt{A^2 + B^2}} = \frac{|2 + 2(-2) - 6|}{\sqrt{1 + 4}} = \frac{|2 - 4 - 6|}{\sqrt{5}}$

$= \frac{8}{\sqrt{5}}$

b. $x - 5y + 3 = 0$ $(3, 3)$

$d = \frac{|1(3) + (-5)(3) + 3|}{\sqrt{1 + 25}} = \frac{|3 - 15 + 3|}{\sqrt{26}}$

$= \frac{|6 - 15|}{\sqrt{26}} = \frac{9}{\sqrt{26}}$

2. Find the distance between the two parallel lines.

a. $4x + y - 6 = 0$
$12x + 3y - 14 = 0$

$y = 6$ $(0, 6)$

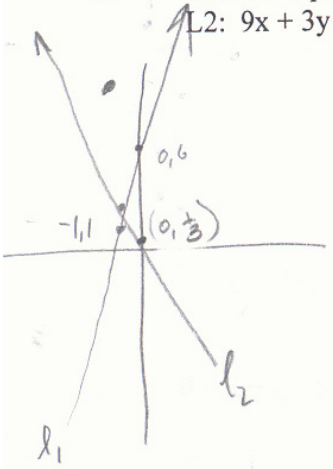
$\frac{|12(0) + 3(6) - 14|}{\sqrt{12^2 + 3^2}} = \frac{|18 - 14|}{\sqrt{144 + 9}} = \frac{4}{\sqrt{153}}$

b. $3x - 2y - 6 = 0$
$3x - 2y - 4 = 0$

$-2y = 6$ $y = -3$ $(0, -3)$

$\frac{|3(0) + (-2)(-3) - 4|}{\sqrt{9 + 4}} = \frac{|6 - 4|}{\sqrt{13}} = \frac{2}{\sqrt{13}}$

3. Find the equation of a bisector of the angle from L1 to L2, given L1 $5x - y + 6 = 0$ and L2: $9x + 3y - 1 = 0$. Include a sketch!



$l_1 = 5x - y + 6 = 0$ $l_2 = 9x + 3y - 1 = 0$

$y = \frac{1 - 9x}{3} = l_2 = -3x + \frac{1}{3}$ (y_1)

$y = 5x + 6 = l_1$ (y_2)

y_1 = solution

$\frac{|5x - y + 6|}{\sqrt{26}} = \frac{|9x + 3y - 1|}{\sqrt{90}}$

$-9.5(5x - y + 6) = 5.1(9x + 3y - 1)$

$-47.5x + 9.5y - 57 = 45.9x + 15.3y - 5.1$

$+47.5 \quad -9.5y \quad +57 \quad +47.5y \quad -9.5y \quad +51$

$93.4x + 5.8y + 51.9 = 0$

$934x + 58y + 519 = 0$

$y = \frac{-519 - 934x}{58}$

4. What geometric property is possessed by all lines of each family? Letters other than x and y are parameters.

a. $y = -\frac{1}{2}x + b$
All lines w/ $m = -\frac{1}{2}$

b. $y = mx - 2$
all lines w/ y-int = -2

c. $y + 5 = m(x + 7)$
lines through $(-7, -5)$

d. $\{y - 4 = m(x - 5) \mid m \text{ real}\}$
all lines thru $(5, 4)$ except $x = 5$

e. $\left\{\frac{x}{a} + \frac{y}{4} = 1 \mid a \text{ real}, a \neq 0\right\}$
all lines thru y intercept 4 except thru the origin

5. Write the GENERAL FORM of the equation of the family of lines possessing the given property.

a. The sum of the intercepts is equal to 7. $\left\{ \frac{x}{a} + \frac{y}{7-a} = 1 \;\middle|\; a \text{ real}, a \neq 0, a \neq 7 \right\}$ DNE $x = 7$ or $y = 7$

b. The product of the intercepts is equal to 6. $\left\{ \frac{x}{a} + \frac{y}{\frac{6}{a}} = 1 \;\middle|\; a \text{ real}, a \neq 0, \right\}$

6. Find an equation(s) of the line(s) containing the point of intersection of $4x + 5y - 1 = 0$ and $3x - 2y + 1 = 0$ and the point (1, 1).

$4x + 5y - 1 = 0$

$15x - 10y + 5 = 0$

$8x + 10y - 2 = 0$

$23x + 3 = 0$

$23x = -3$

$x = -\frac{3}{23}$

$4\left(-\frac{3}{23}\right) + 5y - 1 = 0$ $\left(-\frac{3}{23}, \frac{7}{23}\right)$ $(1, 1)$

$-\frac{12}{23} + 5y - 1 = 0$

$5y - \frac{35}{23} = 0$

$5y = \frac{35}{23}$ $y = \frac{7}{23}$

7. Find an equation(s) of the line(s) that has an x-intercept of 6 and is a distance of 4 from point (2, 4).

8. Find the lengths of the altitudes of the triangle with vertices A(1, 4), B(3, 1), and C(-1, -2). Also find the area of this triangle.

9. Find the lines containing (2, -3) that are (a) parallel and (b) perpendicular to $3x + y + 2 = 0$.

10. For what value(s) of *m* is the line $y = mx + 4$ at a distance of 4 from (-4, -1)?

⑥ $\left(\frac{-3}{23}, \frac{2}{23}\right)$ $(1,1)$

$$\frac{1-\frac{2}{23}}{1-\frac{-3}{23}} = \frac{\frac{16}{23}}{\frac{26}{26}} \quad \frac{16}{26}\cdot\frac{23}{26} = \frac{16}{26} = \frac{8}{13} = m$$

$y - y_1 = m(x - x_1)$

$y - 1 = \frac{8}{13}(x-1)$

$13\left(y-1 = \frac{8}{13}x - \frac{8}{13}\right)$

$13y - 13 = 8x - 8$

$8x - 13y + 5 = 0$

$13y = 8x + 5$

$y = \frac{8}{13}x + \frac{5}{13}$

$(-.1304, .3043)$

y_1

$4x + 5y - 1 = 0$

$5y = 1 - 4x$

$y = -\frac{4}{5}x + \frac{1}{5}$

y_2

$3x - 2y + 1 = 0$

$2y = 3x + 1$

$y = \frac{3}{2}x + \frac{1}{2}$

⑦ $(6,0)$ $d = 4$ from $P = (2,4)$ P119 Ex 7

family: $\{y = m(x-6) \mid m \text{ real}\} \cup \{x = 6\}$

$y = mx - 6m$

$mx - y - 6m = 0 \Rightarrow$ general form of all of those lines

the distance from this line is

$$\frac{|m(2) + (-1)(4) - 6m|}{\sqrt{m^2+1}} = 4$$

$$\frac{|2m - 4 - 6m|}{\sqrt{m^2+1}} = 4$$

$$\frac{|-4m - 4|}{\sqrt{m^2+1}} = 4$$

$-4m - 4 = 4\sqrt{m^2+1}$

$16m^2 + 32m + 16 = 16m^2 + 16$

$32m = 0$

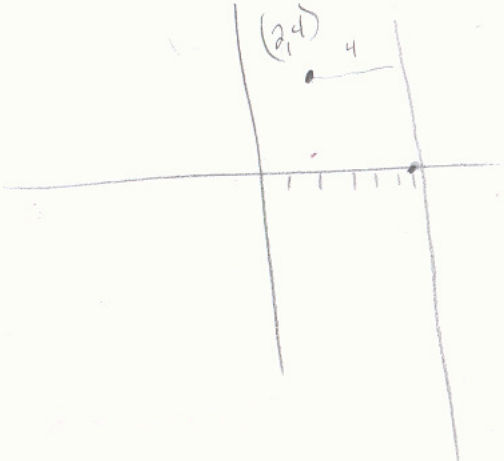
$m = 0$

$y = m(x-6)$

$y = 0(x-6)$

$y = 0$

$x - 6 = 0$



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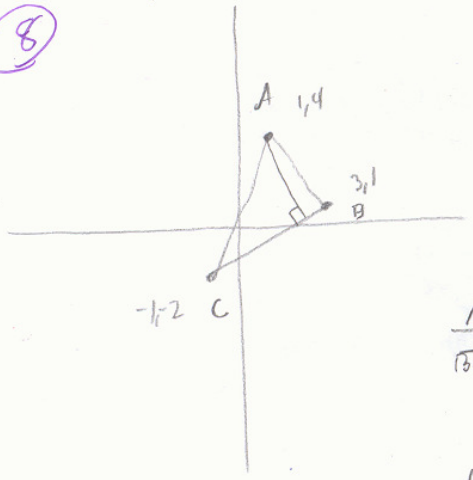


$m_{AB} = \frac{4-1}{1-3} = -\frac{3}{2}$

$m_{\overline{BC}} = \frac{1-(-2)}{3-(-1)} = \frac{3}{4}$

$m_{AC} = \frac{4-(-2)}{1-(-1)} = \frac{6}{2} = 3$

AH. A

$\overline{BC}: m = \frac{3}{4}$

$P = (3,1)$

$P_2 = (1,4)$

$d = \frac{|3(1)+(-4)(4)-5|}{\sqrt{9+16}}$

$d = \frac{|3-16-5|}{\sqrt{25}}$

$d = \frac{|3-21|}{5}$

$d = \frac{18}{5}$

$d = 3.6$

Equation $\overline{BC}$: $y-1 = \frac{3}{4}(x-3)$

$\left(y-1 = \frac{3}{4}x - \frac{9}{4}\right)4$

$4y-4 = 3x-9$

$3x-4y-5 = 0$

AH. A

$m = -\frac{4}{3}$ $P = (1,4)$

$y-4 = -\frac{4}{3}(x-1)$

$\left(y-4 = -\frac{4}{3}x + \frac{4}{3}\right)3$

$3y-12 = -4x+4$

$4x+3y-16 = 0$

AH. B

$P_1 = (3,1)$

Equation $\overline{AB}$

$m = 3$ $P = (1,4)$

$y-4 = 3(x-1)$

$y-4 = 3x-3$

$3x-y+1 = 0$

$d = \frac{|3(3)+(-1)(1)+1|}{\sqrt{9+1}}$

$= \frac{|9|}{\sqrt{10}}$

$= \frac{9}{\sqrt{10}}$

$= 2.85$

AH. C

$m_{AB} = -\frac{3}{2}$ $P = (3,1)$

Equation AB: $y-1 = -\frac{3}{2}(x-3)$

$2\left(y-1 = -\frac{3}{2}x + \frac{9}{2}\right)$

$2y-2 = -3x+9$

$AB = 3x+2y-11 = 0$

$C = (-1,-2)$

$d = \frac{|3(-1)+(2)(-2)-11|}{\sqrt{9+4}}$

$= \frac{|-3-4-11|}{\sqrt{13}}$

$= \frac{|-18|}{\sqrt{13}}$

$= \frac{18}{\sqrt{13}}$

$= 4.992$

P108 Ex 2

(10) values of m $y = mx + 4$ $d = 4$ from $(-4, -1)$

I. General form = $mx - y + 4 = 0$

$$d = \frac{|m(-4) + (-1)(-1) + 4|}{\sqrt{m^2 + 1}}$$

$$d = \frac{|-4m + 1 + 4|}{\sqrt{m^2 + 1}}$$

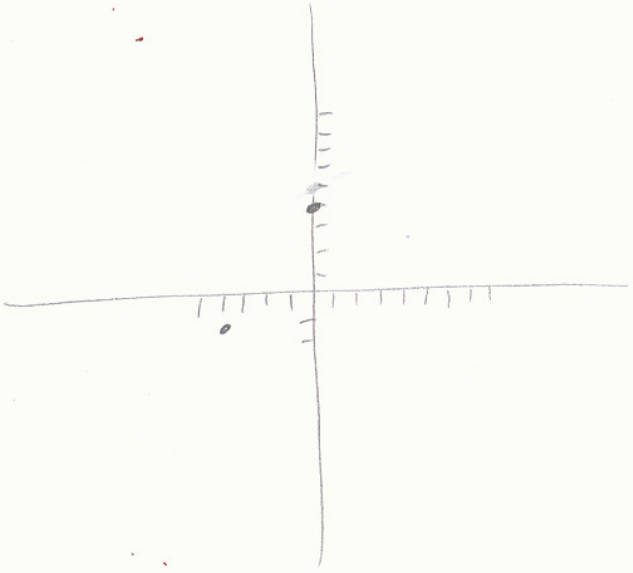
$$4 = \frac{|-4m + 5|}{\sqrt{m^2 + 1}}$$

$$4\left(\sqrt{m^2+1}\right) = -4m + 5$$

$$16m^2 + 16 = 16m^2 - 40m + 25$$

$$40m = 9$$

$$m = \frac{9}{40}$$



$$\left(y = \frac{9}{40}x + 4\right)40$$

$$40y = 9x + 160$$

$$9x - 40y + 160 = 0 \qquad P = (-4, -1)$$

$$d = \frac{|9(-4) + (-40)(-1) + 160|}{\sqrt{81 + 1600}}$$

$$d = \frac{|-36 + 40 + 160|}{\sqrt{1681}}$$

$$d = \frac{|164|}{\sqrt{1681}}$$

$$d = 4 \checkmark$$