Equations of lines have been expressed in various forms. Among these are $y = mx + b$ and $\frac{x}{a} + \frac{y}{b} = 1$. Each of these equations has two constants that have geometrical significance. The constants of the first equation are m and b. When definite values are assigned to these letters, a line is completely determined. Other values for these constants determine other lines. Thus the quantities m and b are fixed for any particular line but change from line to line. These letters are called <u>parameters</u>. In the second equation, a and b would be called parameters. A linear equation with one parameter represents lines all with a particular property. For example, the equation $y = 2x + b$ represents a line with slope 2 and y-intercept b. Consider b a parameter that may assume any real value. Since the slope is the same for all values of b, the equation represents a set of parallel lines. The totality of lines thus determined is called a <u>family</u> of lines with infinitely many lines in the family. Sometimes the family of lines does not represent "all" lines. Often, the letter k will represent a parameter that varies. Note, the previously identified equation $y = 2x + b$ can be written as $y = 2x + k$.

<u>Guided Practice</u>

2/122 $\{y = mx - 5 \mid m \text{ real}\}$

$$y = mx - 5$$
$$y = m(x - 0) + (-5)$$

all lines with y-intercept $(0, -5)$
exclude vertical line $x = 0$

4/122 $\{x = ky \mid k \text{ real}\}$

$$x = ky$$
$$ky = x$$
$$y = \frac{1}{k}(x - 0) + 0$$

all lines through the origin $(0, 0)$
exclude horizontal line $y = 0$
[m will never equal 0]

6/122 $\{2x - 3y = k \mid k \text{ real}\}$

$$2x - 3y = k$$
$$2x - k = 3y$$
$$3y = 2x - k$$
$$y = \frac{2x - k}{3}$$
$$y = \frac{2x}{3} - \frac{k}{3}$$
$$y = \frac{2}{3}x + \left(-\frac{k}{3}\right)$$

all lines with slope $\frac{2}{3}$

Guided Practice

8/122

$\{y = mx + b \mid m, b \text{ real}\}$

$y = mx + b$

$y = m(x + 0) + b$

all lines in the xy plane
exclude all vertical lines
[m will never be undefined]

10/122

$\{x = k \mid k \text{ real}\}$

$x = k$

[vertical line]

all vertical lines

12/122

$\left\{\dfrac{x}{a} + \dfrac{y}{3-a} = 1 \mid a \text{ real}, a \neq 0, a \neq 3\right\}$

given $\dfrac{x}{a} + \dfrac{y}{b} = 1$ and $b = 3 - a$, then $a + b = 3$ [sum of intercepts equals 3]

if $a = 0$, then $\dfrac{x}{0} + \dfrac{y}{3-0} = 1$ where $\dfrac{x}{0}$ does not exist and $\dfrac{y}{3} = 1$ or $y = 3$ is excluded.

if $a = 3$, then $\dfrac{x}{3} + \dfrac{y}{3-3} = 1$ where $\dfrac{y}{0}$ does not exist and $\dfrac{x}{3} = 1$ or $x = 3$ is excluded.

since $a \neq 0$ and $a \neq 3$, then x-intercept $(a,0) = (0,0)$ and $(3,0)$ are not in play.

since $a \neq 0$ and $a \neq 3$, then y-intercept $(0,b) = (0,0)$ and $(0,3)$ are not in play.

all lines with intercepts sum is 3
exclude vertical line $x = 3$
exclude horizontal line $y = 3$

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<u>Guided Practice</u>

$\boxed{16/122}$ All lines perpendicular to $3x-5y-7=0$

given $3x-5y-7=0$, then $m=-\frac{A}{B}=-\frac{3}{-5}=\frac{3}{5}$

since $m_{\perp}=-\frac{5}{3}$, then $A=5, B=3$ and $5x+3y-C=0$ or $5x+3y=k$.

$$\boxed{\{5x+3y=k \mid k \text{ real}\}}$$

$\boxed{18/122}$ All lines with $x-$intercept twice the $y-$intercept

since $a=2b$ and $\frac{x}{a}+\frac{y}{b}=1$, then $\frac{x}{2b}+\frac{y}{b}=1$

since $a=2b$ and $b\neq 0$, then $a\neq 0$ and $(0,0)$ is out of play

so all lines through the origin must be included or

$y=mx+0$ or $y=mx$ or $y=kx$

$$\boxed{\left\{\frac{x}{2b}+\frac{y}{b}=1 \mid b \text{ real},\ b\neq 0\right\}\cup\{y=kx \mid k \text{ real},\ k\neq 0\}}$$