

Equations of lines have been expressed in various forms. Among these are $y = mx + b$ and $\frac{x}{a} + \frac{y}{b} = 1$. Each of these equations has two constants that have geometrical significance. The constants of the first equation are m and b . When definite values are assigned to these letters, a line is completely determined. Other values for these constants determine other lines. Thus the quantities m and b are fixed for any particular line but change from line to line. These letters are called parameters. In the second equation, a and b would be called parameters. A linear equation with one parameter represents lines all with a particular property. For example, the equation $y = 2x + b$ represents a line with slope 2 and y -intercept b . Consider b a parameter that may assume any real value. Since the slope is the same for all values of b , the equation represents a set of parallel lines. The totality of lines thus determined is called a family of lines with infinitely many lines in the family. Sometimes the family of lines does not represent “all” lines. Often, the letter k will represent a parameter that varies. Note, the previously identified equation $y = 2x + b$ can be written as $y = 2x + k$.

Guided Practice

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$\{y = mx - 5 \mid m \text{ real}\}$

$$y = mx - 5$$

$$y = m(x - 0) + (-5)$$

all lines with y -intercept $(0, -5)$
exclude vertical line $x = 0$

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$\{x = ky \mid k \text{ real}\}$

$$x = ky$$

$$ky = x$$

$$y = \frac{1}{k}(x - 0) + 0$$

all lines through the origin $(0, 0)$
exclude horizontal line $y = 0$
[m will never equal 0]

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$\{2x - 3y = k \mid k \text{ real}\}$

$$2x - 3y = k$$

$$2x - k = 3y$$

$$3y = 2x - k$$

$$y = \frac{2x - k}{3}$$

$$y = \frac{2x}{3} - \frac{k}{3}$$

$$y = \frac{2}{3}x + \left(-\frac{k}{3}\right)$$

all lines with slope $\frac{2}{3}$

Guided Practice

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$$\{y = mx + b \mid m, b \text{ real}\} \quad y = mx + b$$

$$y = m(x + 0) + b$$

all lines in the xy plane
exclude all vertical lines
[m will never be undefined]

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$$\{x = k \mid k \text{ real}\} \quad x = k$$

[vertical line]

all vertical lines

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$$\left\{ \frac{x}{a} + \frac{y}{3-a} = 1 \mid a \text{ real}, a \neq 0, a \neq 3 \right\}$$

given $\frac{x}{a} + \frac{y}{b} = 1$ and $b = 3 - a$, then $a + b = 3$ [sum of intercepts equals 3]

if $a = 0$, then $\frac{x}{0} + \frac{y}{3-0} = 1$ where $\frac{x}{0}$ does not exist and $\frac{y}{3} = 1$ or $y = 3$ is excluded.

if $a = 3$, then $\frac{x}{3} + \frac{y}{3-3} = 1$ where $\frac{y}{0}$ does not exist and $\frac{x}{3} = 1$ or $x = 3$ is excluded.

since $a \neq 0$ and $a \neq 3$, then x-intercept $(a, 0) = (0, 0)$ and $(3, 0)$ are not in play.

since $a \neq 0$ and $a \neq 3$, then y-intercept $(0, b) = (0, 0)$ and $(0, 3)$ are not in play.

all lines with intercepts sum is 3
exclude vertical line $x = 3$
exclude horizontal line $y = 3$

Guided Practice

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All lines perpendicular to $3x - 5y - 7 = 0$

$$\text{given } 3x - 5y - 7 = 0, \text{ then } m = -\frac{A}{B} = -\frac{3}{-5} = \frac{3}{5}$$

$$\text{since } m_{\perp} = -\frac{5}{3}, \text{ then } A = 5, B = 3 \text{ and } 5x + 3y - C = 0 \text{ or } 5x + 3y = k.$$

$$\{5x + 3y = k \mid k \text{ real}\}$$

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All lines with x-intercept twice the y-intercept

$$\text{since } a = 2b \text{ and } \frac{x}{a} + \frac{y}{b} = 1, \text{ then } \frac{x}{2b} + \frac{y}{b} = 1$$

since $a = 2b$ and $b \neq 0$, then $a \neq 0$ and $(0,0)$ is out of play

so all lines through the origin must be included or

$$y = mx + 0 \text{ or } y = mx \text{ or } y = kx$$

$$\left\{ \frac{x}{2b} + \frac{y}{b} = 1 \mid b \text{ real, } b \neq 0 \right\} \cup \{y = kx \mid k \text{ real, } k \neq 0\}$$