The <u>Distance from a Point to a Line</u> given the point (x_1, y_1) not on the line $Ax + By + C = 0$ is:

$$d = \frac{Ax_1 + By_1 + C}{\pm\sqrt{A^2 + B^2}} \quad [\text{denominator is given the sign of B where } B \neq 0]$$

The <u>Directed Distance of a Point off a Line</u> is determined by looking at the value of d:

If $d < 0$ or negative, then the point is BELOW a horizontal or slant line or to LEFT of a vertical line.

If $d = 0$ or zero, then the point is ON a horizontal, slant, or vertical line.

If $d > 0$ or positive, then the point is ABOVE a horizontal or slant line or to RIGHT of a vertical line.

<u>Guided Practice</u>

2/112 $\quad 2x - 4y + 2 = 0,\ (1,3) \qquad A = 2,\ B = -4,\ C = 2,\ x = 1,\ y = 3$

$$d = \frac{Ax_1 + By_1 + C}{\pm\sqrt{A^2 + B^2}}$$

$$d = \frac{(2)(1) + (-4)(3) + 2}{-\sqrt{(2)^2 + (-4)^2}}$$

denominator $(-)$ since B is $(-)$

$$d = \frac{2 - 12 + 2}{-\sqrt{4 + 16}}$$

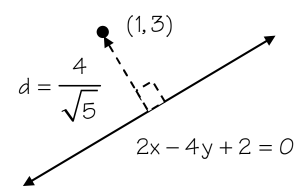
$$d = \frac{-10 + 2}{-\sqrt{20}}$$

$$d = \frac{-8}{-\sqrt{20}}$$

$$d = \frac{8}{2\sqrt{5}}$$

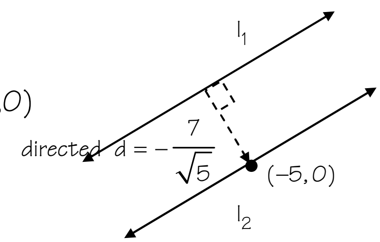
$$d = \boxed{\frac{4}{\sqrt{5}}} \approx 1.79$$

directed d is positive $(+)$ implies the point is ABOVE the line.



12/112 $\quad l_1 : x + 2y - 2 = 0; \qquad A = 1,\ B = +2,\ C = -2$

$l_2 : x + 2y + 5 = 0; \qquad$ if $y = 0$, then $x = -5$ or point $(-5, 0)$



$$d = \frac{Ax_1 + By_1 + C}{\pm\sqrt{A^2 + B^2}}$$

$$d = \frac{(1)(-5) + (2)(0) + (-2)}{+\sqrt{(1)^2 + (2)^2}}$$

denominator $(+)$ since B is $(+)$

$$d = \boxed{-\frac{7}{\sqrt{5}}} \quad [\text{directed distance}]$$

$$d = \left| -\frac{7}{\sqrt{5}} \right|$$

$$d = \boxed{\frac{7}{\sqrt{5}}} \approx 3.13 \quad [\text{real distance}]$$

directed d is negative $(-)$ implies point is BELOW the line and therefore line l_1 is ABOVE line l_2

Independent Practice | Page 112 #1-15 odd

Guided Practice

$\boxed{22/112}$ $l_1:\ 8x+15y-5=0;\quad A=8,\quad B=+15,\quad C=-5$

$l_2:\ 5x-12y+1=0;\quad A=5,\quad B=-12,\quad C=1$

point is ABOVE one line and BELOW the other line

$$d_1=-d_2$$

$$\frac{Ax_1+By_1+C}{\pm\sqrt{A^2+B^2}}=-\frac{Ax_1+By_1+C}{\pm\sqrt{A^2+B^2}}$$

$$\frac{(8)(x)+(15)(y)-5}{+\sqrt{(8)^2+(15)^2}}=-\frac{(5)(x)+(-12)(y)+1}{-\sqrt{(5)^2+(-12)^2}}$$

$$\frac{8x+15y-5}{\sqrt{64+225}}=\frac{5x-12y+1}{\sqrt{25+144}}$$

$$\frac{8x+15y-5}{\sqrt{289}}=\frac{5x-12y+1}{\sqrt{169}}$$

$$\frac{8x+15y-5}{17}=\frac{5x-12y+1}{13}$$

$$13[8x+15y-5]=17[5x-12y+1]$$

$$104x+195y-65=85x-204y+17$$

$$\boxed{19x+399y-82=0}$$

$$m_{bisector}=-\frac{A}{B}=-\frac{19}{399}=-\frac{1}{21}$$

point is above or below BOTH lines

$$d_1=d_2$$

$$\frac{Ax_1+By_1+C}{\pm\sqrt{A^2+B^2}}=\frac{Ax_1+By_1+C}{\pm\sqrt{A^2+B^2}}$$

$$\frac{(8)(x)+(15)(y)-5}{+\sqrt{(8)^2+(15)^2}}=\frac{(5)(x)+(-12)(y)+1}{-\sqrt{(5)^2+(-12)^2}}$$

$$\frac{8x+15y-5}{\sqrt{64+225}}=\frac{5x-12y+1}{-\sqrt{25+144}}$$

$$\frac{8x+15y-5}{\sqrt{289}}=\frac{5x-12y+1}{-\sqrt{169}}$$

$$\frac{8x+15y-5}{17}=\frac{5x-12y+1}{-13}$$

$$-13[8x+15y-5]=17[5x-12y+1]$$

$$-104x-195y+65=85x-204y+17$$

$$\boxed{0=189x-9y-48}$$

$$m_{bisector}=-\frac{A}{B}=-\frac{189}{-9}=\frac{21}{1}=21$$

*** Note that the angle bisectors of intersecting lines are perpendicular as proven in geometry ***

$\boxed{28/112}$ $y=mx+5,\quad (0,0),\quad d=4$

$0=mx-y+5$ with $A=m,\quad B=-1,\quad C=5,\quad x_1=0,\quad y_1=0$

$$d=\frac{Ax_1+By_1+C}{\pm\sqrt{A^2+B^2}}$$

$$4=\frac{(m)(0)+(-1)(0)+5}{-\sqrt{(m)^2+(-1)^2}}$$

$$4=\frac{5}{-\sqrt{m^2+1}}$$

$$4=\frac{-5}{\sqrt{m^2+1}}$$

$$4\sqrt{m^2+1}=-5$$

$$\sqrt{m^2+1}=\frac{-5}{4}$$

$$\left[\sqrt{m^2+1}\right]^2=\left[\frac{-5}{4}\right]^2$$

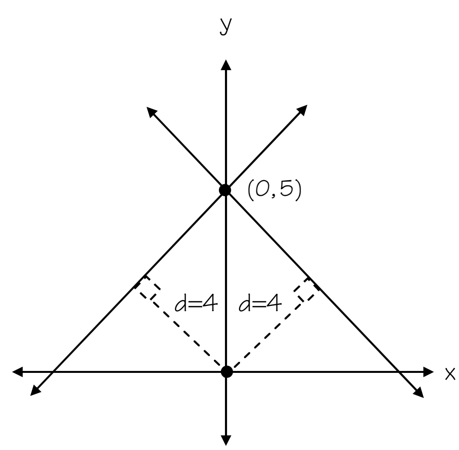
$$m^2+1=\frac{25}{16}$$

$$m^2=\frac{25}{16}-\frac{16}{16}$$

$$m^2=\frac{9}{16}$$

$$m=\pm\sqrt{\frac{9}{16}}$$

$$m=\boxed{\pm\frac{3}{4}}$$



Independent Practice | Page 112 #17-23 odd