

Two simple ways of determining the equation of a line are by one point and the slope or by a pair of points

Point-Slope Form of a line that has slope m and contains the point (x_1, y_1) has the equation:

$$y - y_1 = m(x - x_1) \quad \text{or} \quad y = m(x - x_1) + y_1$$

Two-Point Form of a line through points (x_1, y_1) and (x_2, y_2) where $x_1 \neq x_2$ has the equation:

$$y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1) \quad \text{or} \quad y = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1) + y_1$$

Vertical Line with no slope or $m = \text{DNE}$ and contains the point (a, b) has the equation: $x = a$

Horizontal Line with a slope of zero or $m = 0$ and contains the point (a, b) has the equation: $y = b$

Guided Practice

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Through $(5, 3)$; $m = 4$

one point and the slope implies using the point - slope form

$$y = m(x - x_1) + y_1 \quad \text{point - slope form}$$

$$y = 4(x - 5) + 3$$

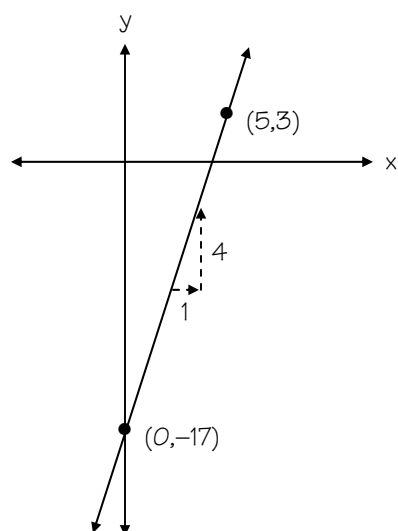
$$y = 4x - 20 + 3$$

$$y = 4x - 17 \quad \text{slope - intercept form}$$

$$17 = 4x - y$$

$$4x - y = 17 \quad \text{standard form}$$

$$4x - y - 17 = 0 \quad \text{general form}$$



using the slope - intercept form of $y = 4x - 17$

$$y = 4x + (-17)$$

$$\text{slope } m = 4 = \frac{4}{1} = \frac{\uparrow 4}{\rightarrow 1} = \frac{\text{rise of 4}}{\text{run of 1}}$$

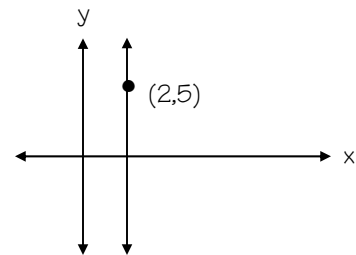
y-intercept of -17 or $(0, -17)$

Guided Practice

8/92

Through (2,5); no slope

no slope implies vertical line through given point

vertical line through (2,5) yields an equation $x = 2$ 

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Through (2,1) and (-3,3)

two points implies using the two-point form

$$y = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1) + y_1 \quad \text{two-point form}$$

$$y = \frac{3 - 1}{-3 - 2}(x - 2) + 1$$

$$y = -\frac{2}{5}(x - 2) + 1$$

$$y = -\frac{2}{5}x + \frac{4}{5} + \frac{5}{5}$$

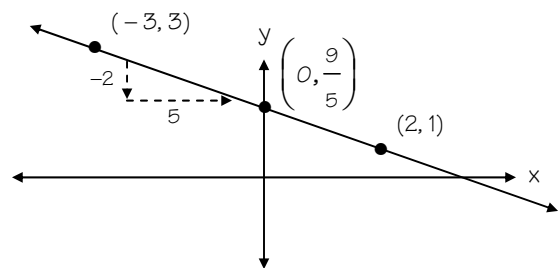
$$y = -\frac{2}{5}x + \frac{9}{5} \quad \text{slope-intercept form}$$

$$(5)[y] = \left[-\frac{2}{5}x + \frac{9}{5}\right](5)$$

$$5y = -2x + 9$$

$$2x + 5y = 9 \quad \text{standard form}$$

$$2x + 5y - 9 = 0 \quad \text{general form}$$

using the slope-intercept form of $y = -\frac{2}{5}x + \frac{9}{5}$

$$\text{slope } m = -\frac{2}{5} = \frac{-2}{5} = \frac{\downarrow 2}{\rightarrow 1} = \frac{\text{rise of } -2}{\text{run of } 5}$$

$$\text{y-intercept of } \frac{9}{5} \quad \text{or} \quad \left(0, \frac{9}{5}\right)$$

Guided Practice**20/92**

$$l_1: x - 5y + 8 = 0, \quad l_2: 4x - y - 6 = 0, \quad l_3: 3x + 4y + 5 = 0$$

 l_1 intersects l_2

$$\begin{array}{r} x - 5y + 8 = 0 \\ 4x - y - 6 = 0 \\ \hline x - 5y + 8 = 0 \\ [-5][4x - y - 6] = [0][-5] \\ \hline x - 5y + 8 = 0 \\ -20x + 5y + 30 = 0 \\ \hline -19x + 0y + 38 = 0 \\ -19x = -38 \\ x = \frac{-38}{-19} \\ x = 2 \end{array}$$

$$\begin{array}{r} x - 5y + 8 = 0 \\ \text{at } x = 2, 2 - 5y + 8 = 0 \\ -5y + 10 = 0 \\ -5y = -10 \\ y = \frac{-10}{-5} \\ y = 2 \end{array}$$

 $(2, 2)$ l_2 intersects l_3

$$\begin{array}{r} 4x - y - 6 = 0 \\ 3x + 4y + 5 = 0 \\ \hline [4][4x - y - 6] = [0][4] \\ 3x + 4y + 5 = 0 \\ \hline 16x - 4y - 24 = 0 \\ 3x + 4y + 5 = 0 \\ \hline 19x + 0y - 19 = 0 \\ 19x = 19 \\ x = \frac{19}{19} \\ x = 1 \end{array}$$

$$\begin{array}{r} 4x - y - 6 = 0 \\ \text{at } x = 1, 4x - y - 6 = 0 \\ 4(1) - y - 6 = 0 \\ -y = 2 \\ y = \frac{2}{-1} \\ y = -2 \end{array}$$

 $(1, -2)$ l_1 intersects l_3

$$\begin{array}{r} x - 5y + 8 = 0 \\ 3x + 4y + 5 = 0 \\ \hline [-3][x - 5y + 8] = [0][-3] \\ 3x + 4y + 5 = 0 \\ \hline -3x + 15y - 24 = 0 \\ 3x + 4y + 5 = 0 \\ \hline 0x + 19y - 19 = 0 \\ 19y = 19 \\ y = \frac{19}{19} \\ y = 1 \end{array}$$

$$\begin{array}{r} x - 5y + 8 = 0 \\ \text{at } y = 1, x - 5(1) + 8 = 0 \\ x - 5 + 8 = 0 \\ x + 3 = 0 \\ x = -3 \end{array}$$

 $(-3, 1)$ **26/92**

$$c_1: x^2 + y^2 + 2x - 19 = 0, \quad c_2: x^2 + y^2 - 10x - 12y + 41 = 0$$

 c_1 intersects c_2

$$\begin{array}{r} x^2 + y^2 + 2x + 0y - 19 = 0 \\ x^2 + y^2 - 10x - 12y + 41 = 0 \\ \hline x^2 + y^2 + 2x + 0y - 19 = 0 \\ [-1][x^2 + y^2 - 10x - 12y + 41] = [0][-1] \\ \hline x^2 + y^2 + 2x + 0y - 19 = 0 \\ -x^2 - y^2 + 10x + 12y - 41 = 0 \\ \hline 12x + 12y - 60 = 0 \\ 12[x + y - 5] = 0 \\ \hline x + y - 5 = 0 \end{array}$$